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LETTER TO THE EDITOR

On the two-parameter theory of solitons in magnetic systems

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Abstract. We demonstrate the inconsistence of the method that introduces two parameters in the study of solitons in magnetic systems: the inverse of the spin magnitude and the characteristic soliton length.

Quite recently, Huang and co-workers have published a series of papers [1–3] in which they present their studies of solitons in one-dimensional ferromagnets, for which they introduced two parameters whose relationship is then assumed. The parameters are $\varepsilon = 1/\sqrt{S}$, where S is the spin magnitude, and η, the characteristic soliton length. The idea was that there exists a certain relationship between these parameters, so the non-linear equation strictly depends upon it. We wish to show that such an approach is inconsistent, and explain the origin of its inconsistence.

The general idea is based on the application of boson formalism. The starting point is the Holstein–Primakoff representation for spin operators [4] which, written with proper dimension, is

$$S_i^z = \hbar(S - a_i^+ a_i) \tag{1a}$$

$$S_i^+ = \hbar(2S - a_i^+ a_i)^{1/2} a_i \tag{1b}$$

$$S_i^- = (S_i^+)^+. \tag{1c}$$

Here a_i are Bose operators

$$[a_i, a_j^+] = \delta_{i,j} \qquad [a_i, a_j] = [a_i^+, a_j^+] = 0. \tag{2}$$

Huang *et al* [1–3] use the expansion in terms of the dimensionless parameter $\varepsilon = 1/\sqrt{S}$ to obtain

$$S_i^+ = S\hbar\sqrt{2}[\varepsilon a_i - \tfrac{1}{4}\varepsilon^3 a_i^+ a_i a_i - \tfrac{1}{32}\varepsilon^5 a_i^+ a_i a_i^+ a_i a_i + \mathrm{O}(\varepsilon^7) \tag{3a}$$

$$S_i^- = (S_i^+)^+. \tag{3b}$$

They obtain the expression for the Hamiltonian:

$$\tilde{H} = \frac{H - H_0}{JS^2\hbar^2} = \varepsilon^2 \left\{ \tilde{f} \sum_i a_i^+ a_i + \tfrac{1}{2} \sum_{i,\delta} [(1+\tau)(a_i^+ a_i + a_{i+\delta}^+ a_{i+\delta}) - a_i a_{i+\delta}^+ - a_{i+\delta} a_i^+] \right\}$$
$$+ \tfrac{1}{8}\varepsilon^4 \sum_{i,\delta} \{[a_i a_{i+\delta}^+ a_{i+\delta}^+ a_{i+\delta} + a_i^+ a_i a_i a_{i+\delta}^+ + \mathrm{HC}] - 4(1+\tau) a_i^+ a_i a_{i+\delta}^+ a_{i+\delta}\}$$

$$+ \tfrac{1}{64}\varepsilon^6 \sum_{i,\delta} [a_i a_{i+\delta}^+ a_{i+\delta}^+ a_{i+\delta} a_{i+\delta}^+ a_{i+\delta} + a_i^+ a_i a_i^+ a_i a_i a_{i+\delta}^+$$

$$- 2a_i^+ a_i a_{i+\delta}^+ a_{i+\delta}^+ a_{i+\delta} + \text{HC}] + O(\varepsilon^8) \tag{4}$$

and calculate the equations of motion for a_i. The equations are formulated in terms of normally ordered products and then averaged over Glauber's coherent states [5]:

$$|\alpha\rangle = \prod_i |\alpha_i\rangle \qquad a_i|\alpha_i\rangle = \alpha_i|\alpha_i\rangle \tag{5}$$

in order to obtain the equations of motion for coherent amplitudes α_i. The continuum approximation is applied up to the fourth power of the lattice constant a and the second dimensionless parameter: $\eta = a/\lambda_0$ is introduced, where λ_0 is some typical wave property, in this case soliton width. Introducing the dimensionless coordinate $x = x/\lambda_0$ and dimensionless time $t = \omega_0 t$ (ω_0 is the typical wave frequency), they obtain their main result:

$$i\bar{\omega}_0\, \partial\alpha/\partial\bar{t} = \varepsilon^2[(\bar{f} + 2\tau)\alpha - \eta^2\alpha_{xx} - \tfrac{1}{12}\eta^4\alpha_{xxxx} + O(\eta^6)]$$

$$+ \varepsilon^4\{-2\tau|\alpha|^2\alpha + \eta^2[-\alpha|\alpha_x|^2 - \tfrac{1}{2}\alpha^2\alpha_{xx}^* + \tfrac{1}{2}\alpha^*(\alpha_x)^2$$

$$- \tau\alpha(|\alpha|^2)_{xx}] + O(\eta^4)\} + \varepsilon^6[\tfrac{1}{4}|\alpha|^2\alpha + O(\eta^2)] + O(\varepsilon^8). \tag{6}$$

Here $\bar{\omega}_0 = h\omega_0/JS^2\hbar^2$.

This equation is now solved under the assumption that there exists a certain relation between η and ε, so all the terms of the same order are retained. The justification is made by analogy with the theory of long waves in shallow water.

We wish to show here that this approach is inconsistent by analysing a simple example. Let us look at the anisotropic model $\tau \neq 0$ and suppose that $\eta = U_1\varepsilon$ where $U_1 = O(1)$. The lowest order non-linear effects appear in the theory of order ε^4. Keeping only these terms in (6) and writing the latter in terms of dimensional variables, we have

$$i\alpha_i = (\mu f + 2\tau JS_c)\alpha - JS_c a^2\alpha_{xx} - (1/S)2JS_c\tau|\alpha|^2\alpha. \tag{7}$$

Notice that here $S_c = S\hbar$, while S also appears, so we encounter a combination of classical and quantum terms. The soliton solution of this equation exists for $\tau > 0$, and is:

$$\alpha = (Sa^2/\tau)^{1/2}\nu \operatorname{sech} \nu(x - x_0 + 2JS_c a^2 kt) \exp[i(kx - \omega t - \varphi_0)] \tag{8}$$

with

$$\omega = \mu f + 2\tau JS_c + JS_c a^2(\nu^2 - k^2). \tag{9}$$

In order to find a relationship between $\lambda_0 = 1/\nu$ and ε, we must use some kind of normalization. If we apply 'naive normalization':

$$\frac{1}{a}\int |\alpha|^2\, dx = 1 \tag{10}$$

we arrive at

$$\lambda_0 = 2Sa/\tau \tag{11}$$

which implies $\lambda_0 \approx \varepsilon^2$, obviously contradicting the initial assumption.

A more physical approach is to suppose that the total magnetization M_z is fixed. We shall treat the magnetization as the deviation of the total z-projection from its maximal value:

$$M_z = \sum_j (S\hbar - S_j^z) = \sum_j \hbar|\alpha_j|^2 = (S_c/S) \sum_j |\alpha_j|^2. \tag{12}$$

In the continuum limit

$$M_z = \frac{S_c}{S}\frac{1}{a}\int |\alpha|^2 \, \mathrm{d}x \tag{13}$$

giving

$$\lambda_0 = (2a/\tau)S_c/M_z. \tag{14}$$

A proper estimate can be made by going back to the semi-classical limit: $S_c = S\hbar$ and $M_z = m\hbar$:

$$\lambda_0 = (2a/\tau)S/m. \tag{15}$$

We notice that m ranges from 0 up to $2NS$, so it is impossible to establish a direct relation between λ_0 and S (or ε).

This was just a simple demonstration of the inconsistency. Its origin lies in the improper use of the expansion (3). The proper procedure has been explained in our papers [6, 7] but we shall outline it here briefly. In this kind of calculation, one must first establish the classical limit $S \to \infty$, $\hbar \to 0$, $S\hbar \to S_c$ and only after that look for the quantum corrections of order $1/S$ or higher.

Let us now look at the expression for S_j^z:

$$S_j^z = S\hbar - \hbar|\alpha_j|^2. \tag{16}$$

In the classical limit S_j^z must be finite, and so $S\hbar \to S_c$, but the last term is the product of a vanishing quantity ($\hbar$) and $|\alpha_j|^2$. It is obvious that $|\alpha_j|^2$ should diverge like S in order to keep this term finite, and that is the reason for our formulating our theory in terms of

$$\tilde{\alpha}_j = \alpha_j/\sqrt{S} = \varepsilon\alpha_j \tag{17}$$

because now we have

$$S_j^z = S_c(1 - |\tilde{\alpha}_j|^2) \tag{18}$$

where $|\tilde{\alpha}_j|^2$ is a finite quantity. The expansion (3) is valid when formulated in terms of $\tilde{\alpha}_j$. We have also shown that taking the classical limit implies neglecting the terms coming from normal ordering, so the expression for H in which the power of ε corresponds to the number of αs that it multiplies is just the classical expression obtained by expanding the square root. This simply means that the general idea of connecting λ_0 and ε is misleading in this approach because most of the εs in [1–3] should not be there at all.

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